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INSTITUTE FOR COMPUTER APPLICATIONS IN SCIENCE AND ENGINEERING  
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## INTRODUCTION

The Institute for Computer Applications in Science and Engineering (ICASE) is operated at the Langley Research Center of NASA by the Universities Space Research Association (USRA) under a contract with the Center. USRA is a non-profit consortium of major U. S. colleges and universities.

The Institute conducts unclassified basic research in applied mathematics, numerical analysis, and computer science in order to extend and improve problem-solving capabilities in science and engineering, particularly in aeronautics and space.

ICASE has a small permanent staff. Research is conducted primarily by visiting scientists from universities and from industry, who have resident appointments for limited periods of time, and by consultants. Members of NASA's research staff also may be resident at ICASE for limited periods.

The major categories of the current ICASE research program are:

- a. Numerical methods, with particular emphasis on the development and analysis of basic numerical algorithms;
- b. Control and parameter identification problems, with emphasis on effective numerical methods;
- c. Computational problems in engineering and the physical sciences, particularly fluid dynamics, acoustics, structural analysis, and chemistry;
- d. Computer systems and software, especially vector and parallel computers, microcomputers, and data management.

ICASE reports are considered to be primarily preprints of manuscripts that have been submitted to appropriate research journals or which are to appear in conference proceedings. A list of these reports for the period October 1, 1982, through March 31, 1983, follows a brief description of research in progress.

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## **RESEARCH IN PROGRESS**

### **PARALLEL SYSTEMS DESIGN METHODOLOGY**

Loyce Adams, Piyush Mehrotra, Merrell Patrick (Duke University), Terrence Pratt (University of Virginia), John Van Rosendale, and Robert Voigt

This is an ongoing group effort to design a parallel architecture for finite element analysis. A top-down design methodology is being used, wherein the entire system is considered in terms of layers of virtual machines. Each of the four layers -- (1) end user's level, (2) researcher's level, (3) operating system level, and (4) the hardware -- is to be formally defined during the design process. The requirements at each level drive the design at the lower level, while the choices at the lower level have an impact on the design of the higher level. Of particular interest are the design of the operating system level and the language features and constructs to be provided to the researcher at the second level.

### **PARALLELISM IN THE SUBSTRUCTURING TECHNIQUE**

Loyce Adams and Robert Voigt

Substructuring is a technique used by structural engineers to subdivide a complex structure into a union of many simple ones. The impact of this approach on the underlying linear system arising from finite element discretization of the structure is that the matrix takes on a block form in which diagonal blocks are related to other diagonal blocks only through common boundary nodes. It turns out that these diagonal blocks may be processed in parallel; and thus, substructuring may be viewed as a mechanism for introducing parallelism into the solution process. The effectiveness of this approach is being studied in the context of the methodology described above.

## **FLOW THROUGH A PIPE**

M. Hanif Chaudhry

One-dimensional transient flow of a slightly compressible liquid through a pipe is described by a set of hyperbolic partial differential equations having a nonlinear source term that represents the friction losses. Shock may be produced at the boundaries due to sudden changes in flow. A number of second-order accurate explicit finite difference methods - MacCormack, Gabutti, Lambda - and MacCormack's fourth-order method were used to integrate these equations numerically.

## **FREE-SURFACE FLOWS**

M. Hanif Chaudhry

Two-dimensional, unsteady, free-surface flows downstream of a partially breached dam are described by a set of hyperbolic partial differential equations having nonlinear source terms. A strong shock or bore is produced at the time of breach. Investigations are in progress to integrate these equations numerically and to study procedures for the simulation of the boundaries.

## **AN UPWIND FINITE DIFFERENCE METHOD FOR TWO-DIMENSIONAL SHOCK CALCULATIONS**

Stephen F. Davis

An ICASE report nearing completion described a first-order upwind scheme which is designed to detect and resolve steady oblique shocks that are not aligned with the computing grid. To accomplish this, the method locates the angle at which a shock might be expected to cross the computing grid and then constructs separate finite difference formulas for the flux components normal and tangential to this direction. Work is in progress to develop a second-order accurate version of this method.

## **A RECURSIVE ALGORITHM FOR HYPERBOLIC EQUATIONS**

Stephen F. Davis

During a visit to ICASE, Professor S. Osher (UCLA) proposed a recursive algorithm for solving Burgers' equation. The basic idea was to solve at discrete spatial points and specified times the recursive formula  $u(x,t) = f(x - u(x,t)t)$ , which characterizes the exact solution. At the initial time,  $f(x)$  is taken to be a spectral interpolation of the initial data; the algorithm can be restarted later by setting  $t = 0$  and  $f(x)$  to a spectral interpolation of the computed solution at that time. Preliminary calculations on Burgers' equation show the method to be fast and accurate. Work is in progress to analyze this algorithm, extend it to hyperbolic systems, and study the possibility of using it in a splitting scheme for multidimensional problems or problems with source terms.

## **AN ADAPTIVE FINITE ELEMENT METHOD FOR INITIAL BOUNDARY VALUE PROBLEMS**

Stephen F. Davis, Joseph E. Flaherty, and R. Ludwig (Rensselaer Polytechnic Institute)

In a continuation of the work reported in ICASE Report 81-13, a study of time-dependent algorithms for moving computing grids has been conducted. It was discovered that many of the proposed algorithms for moving grid points are unconditionally unstable for parabolic equations and only neutrally stable for hyperbolic equations. Some of this work was presented at the ARO Workshop on "Adaptive Numerical Methods for Partial Differential Equations."

## **MICROPROCESSOR RESEARCH**

Stefan Feyock

Intel Corporation has donated two hardware systems to ICASE: an iDBP database processor, and a microprocessor network consisting of two i432 microprocessors as well as associated hardware necessary for network software

development. Research has focused on experimentation with and development of relevant software tools for these systems.

iDBP Research. Work on the database machine has been performed jointly with Paul Fishwick of Kentron, Inc., and is aimed toward the goal of developing a High-Level-Data-Abstraction system (HILDA), a three-layer database management system supporting the data abstraction features of the iDBP. The lowest-level layer, implementing the Service-Port-Protocol (SPP), is essentially complete. Work is currently in progress on the middle layer, an assembly language for constructing iDBP request modules.

1432 Network Research. Progress on this project has been hampered by hardware problems; at this writing, the network is not yet operational. For this reason, work has concentrated on developing software tools relevant to programming the network once it becomes operative. In particular, since it is intended to approach network programming in Ada by using a parser generator (MYSTRO) to do source-to-source transforms from single concurrent programs to multiple sequential programs, investigations have been under way into effective methods for specifying the semantics of such transforms. Encouraging progress has been made along these lines; a semantics specification system based on the developed approach has been implemented and is currently being debugged. Upon completion of this phase the semantics specification system will be integrated into the parser generator and used as a vehicle for network programming.

## **APPLICATION OF NONLINEAR PROGRAMMING TO A LINEAR-QUADRATIC FORMULATION IN CONTROL SYSTEM DESIGN**

Peter Fleming

The familiar suboptimal regulator design approach is recast as a constrained optimization problem and incorporated in a computer-aided design package where both design objective and constraints are quadratic cost functions. This formulation permits the separate consideration of, for



example, model-following errors, sensitivity measures, and control energy as objectives to be minimized or limits to be observed. Efficient techniques for computing the interrelated cost functions and their gradients are utilized in a nonlinear programming algorithm. A number of examples have been studied, and two ICASE reports are in preparation. Future work will concentrate on a related multi-objective optimization problem.

#### **PARALLEL ARCHITECTURES FOR ADAPTIVE ELLIPTIC SOLVERS**

Dennis B. Gannon and John Van Rosendale

Parallel architectures having hundreds or thousands of processors are now feasible and may soon be used for numerical computation. With such computers, subtle problems arise in storage allocation, task synchronization, and extraction of parallelism, which the numerical analyst does not currently face. This research addresses these issues in the context of adaptive iterative algorithms for elliptic partial differential equations. A novel cluster architecture well suited to this class of problem has been designed and simulated. Current work is directed at the software issues involved in the use of such an architecture, focusing especially on data flow ideas which relieve the user of much of the programming burden.

#### **CALCULATION OF SEPARATED FLOW**

Thomas B. Gatski (LaRC) and Chester E. Grosch

A two-dimensional, time-dependent Navier-Stokes code developed previously is being used to calculate separating flows. The purpose of this study is to gain insight into the dynamics of this class of flow as the Reynolds number is varied and to investigate the interaction of small disturbances in the flow with the separation region. The numerical results are being compared with some very recent experimental results.

## **INTERACTION OF FREE STREAM DISTURBANCES WITH THE BOUNDARY LAYER ON SLENDER BODY**

Chester E. Grosch

Work has begun on using a Navier-Stokes code to calculate the flow on slender, blunt-nosed bodies. Small, time-dependent disturbances are introduced into the free stream ahead of the body. The objective of this work is to understand the interaction of these disturbances with the boundary layer, particularly in the leading edge region, and the generation of Tollmien-Schlichting waves in the boundary layer.

## **ITERATIVE METHODS FOR FIRST-ORDER SYSTEMS**

Chester E. Grosch, Timothy N. Phillips, and Milton E. Rose

There exists a rather well-developed methodology for solving elliptic second-order partial differential equations by iterative methods. It is not clear how to adopt this methodology in order to treat general systems of first-order elliptic equations. For this purpose, a study of iterative methods for the Cauchy-Riemann equations is in progress, with particular emphasis being given to multigrid techniques. Results to date show progress when a least-squares formulation is used (cf., Fix and Rose, ICASE Report 83-4). Experiments are being done with multigrid techniques as applied to compact finite difference schemes for the Cauchy-Riemann equations. The results to date show some success, in that there is a decrease in the spectral radius as compared to single grid relaxation. However, a number of problems remain and work is continuing.

## **SMOOTHING FOR PSEUDOSPECTRAL APPROXIMATIONS OF HYPERBOLIC EQUATIONS WITH DISCONTINUOUS SOLUTIONS**

M. Yousuff Hussaini, David Kopriva, and Thomas A. Zang, (LaRC)

The usefulness of Fourier pseudospectral approximations to nonlinear hyperbolic problems with discontinuous solutions is being assessed. In particular, filtering strategies for removing oscillations which develop in the solutions are being tested. The goal is to determine if spectral accuracy can be obtained as it can for linear problems.

## **TURBULENCE AMPLIFICATION IN SHOCK-TURBULENCE INTERACTIONS**

M. Yousuff Hussaini, David Kopriva, and Thomas A. Zang (LaRC)

Turbulence amplification and sound generation resulting from the interaction of a plane shock wave with low amplitude turbulence are being studied with a pseudospectral code. The discretization is Chebyshev normal to the shock and Fourier along the shock. For increased accuracy, shock fitting is used. Preliminary results comparing the pseudospectral computations with finite difference computations and linear plane wave theory are described in an ICASE report.

## **SPECTRAL SHOCK-FITTED SOLUTIONS OF TRANSONIC FLOWS PAST A CIRCULAR CYLINDER**

M. Yousuff Hussaini, David Kopriva, M. D. Salas (LaRC), and Thomas A. Zang (LaRC)

A Chebyshev pseudospectral code for the Euler gas dynamics equations is being developed to compute transonic flows over a circular cylinder. The shock is fitted so that in the computational domain the flow is smooth. The goal is to assess the suitability of using spectral methods to compute transonic flows.

## **EFFICIENT ITERATIVE SCHEMES FOR SPECTRAL EQUATIONS**

M. Yousuff Hussaini, Timothy N. Phillips, and Thomas A. Zang (LaRC)

The purpose of this research is to investigate iterative techniques for the solution of systems arising from the application of spectral methods to partial differential equations. Of particular interest are general elliptic problems with Dirichlet/Neumann boundary conditions in which the Chebyshev spectral operation is required. This is a very ill-conditioned problem since the condition number of the operator is  $O(N^4)$ . Various preconditioners have been considered, including higher-order finite differences and several diagonals of the spectral matrix itself. Efficient solution of the resulting problem is obtained using incomplete LU decomposition methods. These preconditioning methods are particularly effective when used in conjunction with multigrid methods.

## **PARAMETER ESTIMATION IN PDEs**

Kazufumi Ito

A technique based upon Legendre-tau approximation has been developed to study one-dimensional seismic inverse problems. Our study consists of estimating unknown parameters in boundary conditions, as well as unknown elastic moduli from observations of displacement at the surface. An ICASE report describing the research is in progress.

## **LEGENDRE-SPECTRAL APPROXIMATION FOR PDEs**

Kazufumi Ito

A study on the use of the Legendre-spectral method for various partial differential equation (e.g., nonlinear-hyperbolic, reaction-diffusion equations) is being considered. Some partial theoretical and numerical results have been obtained for one-dimensional Burgers' equation and two-point boundary value problems.

## **NUMERICAL APPROXIMATION OF LQR FEEDBACK GAINS FOR DIFFERENTIAL-DELAY SYSTEMS**

Kazufumi Ito and Russell Teglus

A numerical method based upon the Legendre-tau approximation has been developed to compute linear-quadratic regulator feedback gains for a variety of linear differential-delay systems. Numerical results indicate that the method is very competitive, both in accuracy and efficiency, when compared with other existing methods. An ICASE report is nearing completion.

## **SOUND GENERATION FROM SHOCK-VORTEX INTERACTIONS**

David A. Kopriva and H. S. Ribner (LaRC)

A detailed study of the cylindrical sound wave generated when a plane shock interacts with a cylindrical vortex is under way. Both linearized analytic and numerical computations are being compared. A model proposed by Ribner which Fourier decomposes the vortex in order to use the theory of shock - plane wave interactions is now being analyzed. Also second- and fourth-order finite difference codes in which shock fitting is used have been developed. Detailed comparisons between these two theoretical approaches and experiments will be made.

## **INTEGRATING MATRICES FOR DIFFERENT EQUATIONS**

Raymond Kvaternik (LaRC) and William D. Lakin

Integrating matrices form the basis of an efficient numerical procedure for solving differential equations associated with clamped-free rotating beam configurations. The current work has generalized integrating matrix methodology to beam configurations with arbitrary boundary conditions. Vibration problems for elastic plates have also been addressed.

## **CONCURRENT MANIPULATION OF DATA STRUCTURES USED IN DATABASES**

Piyush Mehrotra

Multiuser data bases require that the manipulation of the data records be allowed by independent processes in such a manner as to preserve the integrity of the database while avoiding deadlock among the processes.  $B^*$ -trees are one of the many tree structures utilized to implement data bases. Mechanisms, which avoid deadlocks or starvation have been proposed to allow the concurrent retrieval, deletion, and addition of data records to such a tree structure. The feasibility of the mechanisms and their performance can be studied using a simulation of the whole process.

## **LANGUAGE INTERFACE FOR USERS OF LARGE ARRAYS ON PARALLEL ARCHITECTURES**

Piyush Mehrotra

The partitioning and distribution of large arrays on parallel architectures raise special difficulties. The concept of rectangular "windows" had been proposed to allow the user to partition dense matrices among the tasks needed to solve the particular problem at hand. Ongoing investigation is looking at extending the concept to other types of matrices such as banded and sparse. The concept of windows is also being extended to include non-rectangular windows. A model has been implemented and tested as an Ada package. The implementation of the model on the Intel 432, a multiprocessor system, is being investigated.

## **SPIN MODE SOUND RADIATION FROM A THICK WALL DUCT**

William F. Moss

A mathematical model of spin mode sound radiation has been developed using singular integral equations. It is assumed that the spin mode synthesizer isolates a pure azimuthal mode inside the duct and that the form of this interior sound pressure is known up to a sequence of reflection

coefficients. These reflection coefficients and the sound pressure on the exterior duct wall are the unknowns in our integral equation formulation. Once the integral equation has been solved, the far field pattern can be found by quadrature. We have developed methods for the accurate computation of the singular integrals which occur, and we have studied the variation of the condition number of the discretized problem with respect to changes in the wave number.

#### **PARALLEL METHODS FOR PARTIAL DIFFERENTIAL EQUATIONS**

James Ortega (University of Virginia) and Robert Voigt

The review of numerical techniques appropriate for the parallel solution of partial differential equations has continued. Both direct and iterative techniques for a variety of different architectures are considered. An extensive bibliography is being prepared.

#### **NUMERICAL SOLUTION OF A COUPLED PAIR OF ELLIPTIC EQUATIONS FROM SOLID STATE ELECTRONICS**

Timothy N. Phillips

Iterative methods are considered for the solution of a coupled pair of second-order elliptic equations. A finite difference scheme is used which retains the conservative form of the differential equations. Numerical solutions are obtained in two ways - by multigrid and by dynamic ADI methods. Results show that multigrid methods are an effective means of solving this problem. Future work will deal with a more efficient method of treating the boundary conditions within the multigrid context.

## **NATURAL CONVECTION IN AN ENCLOSED CAVITY**

Timothy N. Phillips

We are concerned with the problem of buoyancy-driven flow in a vertical, rectangular cavity whose vertical sides are at different temperatures and whose horizontal sides are insulated. The dynamic ADI method is used to obtain numerical solutions to this problem. For large values of the Rayleigh number, the flow patterns develop strong boundary layers. These boundary layers are resolved using a non-uniform grid.

## **A PRECONDITIONED FORMULATION OF THE CAUCHY-RIEMANN EQUATIONS**

Timothy N. Phillips

A preconditioning of the Cauchy-Riemann equations which results in a second-order system is described. This system is shown to have a unique solution if the boundary conditions are chosen carefully. This choice of boundary conditions enables the solution of the first-order system to be retrieved. Numerical solutions are obtained using the multigrid method.

## **FIRST-ORDER SYSTEMS IN ELASTIC MEDIA**

Timothy N. Phillips and Milton E. Rose

An important example of a first-order system of elliptic equations is the equilibrium equations of elastic media when expressed in terms of stress and displacements. Finite difference methods can complement finite element methods for such problems if fast-solution techniques can be developed and if, at the same time, accurate computations of stresses can be achieved. Some progress has been made in employing multigrid methods by exploiting formulation of the problem in terms of displacements; however, a more direct approach is also being sought.



## **FINITE DIFFERENCE METHOD FOR THE HELMHOLTZ EQUATION**

Milton E. Rose

The Helmholtz equation  $\Delta^2 u + k^2 n^2 u = 0$  (when  $n$  is slowly varying and  $k$  is large) presents a major challenge for solution by finite difference methods. A novel system of finite difference equations has been developed that indicates a truncation error  $O(k^2 h^2)$  when  $kh$  is small and  $O(k^{-2} h^{-2})$  when  $kh$  is large,  $h$  being a representative mesh parameter. The latter is a geometrical optics approximation. These equations form a first-order system, and effective iterative methods for their solution depend upon studies reported elsewhere.

## **OPERATION OF THE VAX-750**

Nancy Shoemaker

The VAX-750 is now a fully operational research tool, running under the UNIX operating system and supporting such activities as symbolic computation and Ada programming, in addition to numerical computation. With the assistance of Chris Wild of Old Dominion University, work is under way to link the VAX to the central computer complex as a remote job entry device. This capability would facilitate the development of programs on the VAX that are to be executed at the central site.

## **WINGS FOR SUPERCRUISER**

Sivagura S. Sritharan

A direct design method has been developed, to obtain conical wings at supersonic speeds with shock-free supercritical cross flow. Results for simple configurations will soon be published.

## **TRANSITION TO TURBULENCE IN INCOMPRESSIBLE FLOWS**

Sivagura S. Sritharan

Theoretical and computational aspects of the early onset of turbulence are being studied. The possibility of using group theoretical concepts to devise symmetry-adapted numerical techniques is being investigated.

## **GEOMETRICAL ASPECTS OF TRANSONIC COMPUTATIONS**

Atef Sherif (George Washington University), S. S. Sritharan, and Russell Teglas

Differential geometric concepts involved in the formulation of the computational procedures for rotational and irrotational compressible flows -- such as finite volume/area concepts and multistream function techniques as well as grid generation methods for curved surfaces -- are being studied.

## **LONG-TIME BEHAVIOR OF THE SCALAR GENUINELY NONLINEAR LAX-FRIEDRICHS SCHEME**

Eitan Tadmor

A study of the Lax-Friedrichs scheme, approximating the scalar, genuinely nonlinear conservation law  $u_t + f_x(u) = 0$ , where  $f(u)$  is, say, strictly convex,  $f''(a_*) > 0$ , is under way. It has been shown that the divided differences of the numerical solution at time  $t$  do not exceed  $2(t a_*'')^{-1}$ . This one-sided Lipschitz boundedness is in complete agreement with the corresponding estimate one has in the differential case; in particular, it is dependent on the initial amplitude, in sharp contrast to linear problems. It guarantees the entropy-satisfying compactness of the scheme in this case and provides a quantitative insight into the long-time behaviour of the numerical computation.

## **STABILITY OF HIGHLY ACCURATE DISCRETE METHODS FOR TIME-DEPENDENT SYSTEMS**

Eitan Tadmor

A unified review for the stability theory of highly accurate methods approximating time-dependent systems is under development. The finite difference, Fourier, and Galerkin methods of lines, applied to the model problem of first-order periodic hyperbolic systems are considered. Emphasis is given to those features which are unique to each of the above methods and which are responsible for their different stability behaviour. In short, the following dilemma regarding high accuracy and aliasing is discussed; in the classical finite difference methods, there is a built-in mechanism to smooth out the aliasing; in the Fourier method, at the present state of the theory, smoothing must be added; and in the Galerkin method, aliasing is completely suppressed.

## **IDENTIFICATION AND CONTROL TECHNIQUES FOR ROBOTIC MANIPULATORS**

Russell Teglus

Research to date in this area has focused on two topics: (1) the off-line identification of various parameters (link inertia, Coulomb friction, etc.) that occur in the equations used to model the dynamics of a manipulator; (2) the development of an adaptive controller which will cause the end-effector of the manipulator to follow, with an unknown load, a given reference trajectory.

## **ADAPTIVE INTERPOLATION AND QUADRATURE OF SINGULAR FUNCTIONS**

John Van Rosendale

Interpolation of nonsmooth data is a problem arising frequently -- for example, in equation of state calculations, in table look up, and in finite element solutions of partial differential equations. Numerical quadrature of singular functions is a closely related problem. Piecewise polynomial approximations converge very slowly for two- and three-dimensional problems

where the data has curves or surfaces of discontinuity. On a uniform grid, piecewise polynomial approximations to such functions are less than first-order convergent. Adaptive refinement improves the situation considerably, but to obtain rapid convergence, adaptive refinement must be combined with algorithms which orient the polynomial patch boundaries to the curves or surface of discontinuity. Theoretical and experimental work on such algorithms is in progress.

## REPORTS AND ABSTRACTS

October 4, 1982 through March 31, 1983

**Banks, H. T. and P. L. Daniel:** *Estimation of variable coefficients in parabolic distributed systems.* ICASE Report No. 82-32, November 1, 1982, 52 pages. Submitted to IEEE Trans. Automat. Control.

We present techniques based on cubic spline approximations for estimating coefficients (e.g., diffusion, convective velocity, etc.) depending on time and the spatial variable in parabolic distributed systems such as those that arise in transport models. Convergence results and a summary of numerical performance of the resulting algorithms are given.

**Tadmor, E.:** *The unconditional instability of inflow-dependent boundary conditions in difference approximations to hyperbolic systems.* ICASE Report No. 82-33, November 10, 1982, 22 pages. To appear in Math. Comp.

It is well known that for a mixed initial-boundary hyperbolic system to be well-defined it is necessary to impose additional boundary conditions only on the inflow eigenspace of the problem. We prove the discrete analogue of the above concerning difference approximations to such system; that is, imposing numerical boundary conditions which are at least zeroth-order accurate with an inflow part of the interior equations, leads to unconditional instability.

**Harten, A.:** *On a large time-step high resolution scheme.* ICASE Report No. 82-34, November 15, 1982, 20 pages. Submitted to Math. Comp.

This paper presents a class of new second-order accurate  $(2K+3)$ -point explicit schemes for the computation of weak solutions of hyperbolic conservation laws, that are under a CFL restriction of  $K$ . These highly nonlinear schemes are obtained by applying a nonoscillatory first-order accurate  $(2K+1)$ -point scheme to a modified flux. The so derived second-order accurate schemes achieve high resolution, while retaining the robustness of the original first-order accurate scheme.

**Osher, S.:** *Smoothing for Spectral Methods.* ICASE Report No. 82-35, November 16, 1982, 12 pages. Proceedings of Spectral Methods Workshop, SIAM/ICASE, Hampton, VA, August 16-18, 1982.

The purpose of this article is mainly to demonstrate, by means of simple examples, that some kind of smoothing must be an essential part of any spectral method. Spectral methods have, in principle, infinite order of

accuracy, if the true solution is smooth. I shall review some old work, done jointly with Majda and McDonough [1], which shows how drastically the situation changes when discontinuities are present. The error pollutes the solution globally if no smoothing is used, and pollutes it in a very large region, even if smoothing based on the finite spectral transform is used. A more drastic type of smoothing will remove this error. Smoothing is, in general, necessary for two reasons:

- a. Accuracy
- b. Stability

**Gannon, D. and J. R. Van Rosendale:** *Highly parallel multigrid solvers for elliptic PDEs: An experimental analysis.* ICASE Report No. 82-36, November 17, 1982, 38 pages. Submitted to SIAM J. Sci. Statis. Comput.

Computer Architectures consisting of many thousands of processing elements have been proposed and studied in the literature. In many cases, the motivation for building these systems is to speed-up large scale scientific computation such as the solution of elliptic partial differential equations. To provide a basis for comparing architectural alternatives, it is helpful to analyze the various classes of algorithms that might be well suited to a highly parallel implementation. This paper considers two existing variations of the Multigrid technique that have been considered suitable for parallel computation and describes a new algorithm which is designed to exploit a greater level of concurrency than the previous schemes. Based on architectures proposed in the literature, a series of interprocessor communication models is developed to serve as a basis for comparing the selected algorithms. The algorithms are tested experimentally and their performance for each communication model is illustrated.

**Friedland, S. and E. Tadmor:** *Optimality of the Lax-Wendroff condition.* ICASE Report No. 82-37, November 23, 1982, 12 pages. To appear in Linear Algebra Appl.

We show that any spectrally dominant vector norm on matrices which is invariant under isometries, dominates the numerical radius,  $r(\cdot)$ . Thus, the celebrated Lax-Wendroff stability condition,  $r(\cdot) \leq 1$ , [4], defines a maximal isometrically invariant stable set.

**Gottlieb, D., L. Lustman, and C. Streett:** *Spectral methods for two-dimensional shocks.* ICASE Report No. 82-38, November 24, 1982, 25 pages. Proceedings of Spectral Methods Workshop, SIAM/ICASE, Hampton, VA, August 16-18, 1982.

Chebyshev collocation methods are applied to two-dimensional gasdynamic problems with shocks, namely inviscid flow (Euler equations) and transonic flow (full potential equation). Boundary conditions, numerical dissipation and iteration techniques are considered, and computational results are presented and compared with finite-difference results.

**Abarbanel, S. S., D. L. Dwyer, and D. Gottlieb:** *Stable implicit finite-difference methods for three-dimensional hyperbolic systems.* ICASE Report No. 82-39, November 26, 1982, 25 pages. Submitted J. Comput. Phys.

The numerical stability of conventional implicit algorithms of the approximate factorization type as applied to two-dimensional and three-dimensional hyperbolic systems is analyzed. The unconditional instability of these methods for the three-dimensional wave equation and Euler's equations of gas dynamics is proven and a modified scheme is proposed which is unconditionally stable for the scalar wave equation. Analytical results are verified by numerical experimentation.

**Hussaini, M. Y. and T. A. Zang:** *Iterative spectral methods and spectral solutions to compressible flows.* ICASE Report No. 82-40, November 30, 1982, 30 pages. Proceedings of Spectral Methods Workshop, SIAM/ICASE, Hampton, VA, August 16-18, 1982.

A spectral multi-grid scheme is described which can solve pseudospectral discretizations of self-adjoint elliptic problems in  $O(N \log N)$  operations. An iterative technique for efficiently implementing semi-implicit time-stepping for pseudospectral discretizations of Navier-Stokes equations is discussed. This approach can handle variable coefficient terms in an effective manner. Pseudospectral solutions of compressible flow problems are presented. These include one-dimensional problems and two-dimensional Euler solutions. Results are given both for shock-capturing approaches and for shock-fitting ones.

**Geer, J. F. and J. C. Strikwerda:** *Vertical slender jets with surface tensions.* ICASE Report No. 82-41, December 16, 1982, 29 pages. Submitted to J. Fluid Mech.

The shape of a vertical slender jet of fluid falling steadily under the force of gravity is studied. The problem is formulated as a nonlinear free boundary value problem for the potential. Surface tension effects are included and studied. The use of perturbation expansions results in a system of equations that can be solved by an efficient numerical procedure. Computations were made for jets issuing from three different orifice shapes, which were an ellipse, a square, and an equilateral

triangle. Computational results are presented illustrating the effects of different values for the surface tension coefficient on the shape of the jet and the periodic nature of the cross-sectional shapes.

**Tadmor, E.:** *Skew-selfadjoint form for systems of conservation laws.* ICASE Report No. 82-42, December 23, 1982, 22 pages. Submitted to J. Math. Anal. Appl.

We study hyperbolic systems of conservation laws augmented with an entropy inequality. We show that such systems can be written in a (quasilinear) skew-selfadjoint form. Centered differencing of such a form under the smooth regime, ends up with a systematic recipe for constructing quasiconservative schemes where the global entropy conservation is recovered. Employing the above formulation in bounded regions under the nonsmooth regime as well, we further conclude a local entropy decay estimate. Examples of the shallow-water and the full gasdynamics equations are explicitly treated.

**Phillips, T. N.:** *Numerical solution of a coupled pair of elliptic equations from solid state electronics.* ICASE Report No. 83-1, January 12, 1983, 18 pages. Submitted to J. Comput. Phys.

Iterative methods are considered for the solution of a coupled pair of second order elliptic partial differential equations which arise in the field of solid state electronics. A finite difference scheme is used which retains the conservative form of the differential equations. Numerical solutions are obtained in two ways - by multigrid and dynamic alternating direction implicit methods. Numerical results are presented which show the multigrid method to be an efficient way of solving this problem.

**Phillips, T. N.:** *Natural convection in an enclosed cavity.* ICASE Report No. 83-2, February 24, 1983, 24 pages. Submitted to J. Comput. Phys.

We are concerned with the problem of buoyancy driven flow in a vertical, rectangular cavity whose vertical sides are at different temperatures and whose horizontal sides are insulated. An application of the dynamic A.D.I. method to obtain numerical solutions to this problem is described. For large non-dimensional temperature differences characterized by the Rayleigh number the flow patterns develop strong boundary layers. These boundary layers are resolved by applying the D.A.D.I. method to the discretization of this problem on a non-uniform grid.



**Dahlburg, R. B., T. A. Zang, D. Montgomery and M. Y. Hussaini:** *Viscous, resistive MHD stability computed by spectral techniques.* ICASE Report No. 83-3, March 1, 1983, 28 pages. To appear Proc. Nat. Acad. Sci.

Expansions in Chebyshev polynomials are used to study the linear stability of one-dimensional magnetohydrodynamic (MHD) quasi-equilibria, in the presence of finite resistivity and viscosity. The method is modeled on the one used by Orszag in accurate computation of solutions of the Orr-Sommerfeld equation. Two Reynolds-like numbers involving Alfvén speeds, length scales, kinematic viscosity, and magnetic diffusivity govern the stability boundaries, which are determined by the geometric mean of the two Reynolds-like numbers. Marginal stability curves, growth rates versus Reynolds-like numbers, and growth rates versus parallel wave numbers are exhibited. A numerical result which appears general in that instability has been found to be associated with inflection points in the current profile, though no general analytical proof has emerged. It is possible that nonlinear subcritical three-dimensional instabilities may exist, similar to those in Poiseuille and Couette flow

**Fix, G. J., and M. E. Rose:** *A comparative study of finite element and finite difference methods for Cauchy-Riemann type equations.* ICASE Report No. 83-4, March 28, 1983, 22 pages. Submitted to SIAM J. Numer. Anal.

A least squares formulation of the system  $\text{div} \underline{u} = \rho$ ,  $\text{curl} \underline{u} = \underline{\zeta}$  is surveyed from the viewpoint of both finite element and finite difference methods. Closely related arguments are shown to establish convergence estimates.

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## ICASE SEMINAR PROGRAM

October 4, 1982 through March 31, 1983

- October 12 **Dr. James M. Hyman**, Los Alamos Scientific Laboratory: Adaptive Mesh Strategies for the Numerical Solution of Differential Equations
- October 13 **Ms. Debby Hyman**, Los Alamos Scientific Laboratory: The Los Alamos Integrated Graphics System
- October 21 **Dr. William Poole**, Boeing Computer Services Company: A Preliminary Look at Numerical Computations Using the Fujitsu Facom VP-100 and VP-200 Vector Processors
- October 22 **Dr. Peter Fleming**, Syracuse University: Suboptimal Controller Design
- October 25 **Professor Maurice Holt**, University of California, Berkeley: The Application of Glimm's Method to Surface Wave Problems
- November 18 **Dr. John Van Rosendale**, ICASE: Efficient Path Counting and Network Reliability Estimation on Sparse Graphs
- November 22 **Dr. Ilse Ipsen**, The Pennsylvania State University: Stable Matrix Computations in VLSI
- December 1 **Dr. M. D. Salas**, NASA Langley Research Center: Transonic Flow Calculations: Can They Be Trusted?
- December 2 **Professor Robert A. Wagner**, Duke University: The Boolean Vector Machine (BVM)
- December 13 **Professor Steven Dubowsky**, Massachusetts Institute of Technology: On The Dynamics and Control of Robotic Manipulators
- December 15 **Professor James Geer**, State University of New York: Symbolic Computation and Applied Mathematics
- January 14 **Dr. Paul Woodward**, Lawrence Livermore National Laboratory: The Piecewise-Parabolic Method: A Hydrodynamics Scheme for Fluid Flow with Strong Shocks
- January 17 **Professor Maurizio Pandolfi**, Politecnico di Torino, Italy: A Contribution to the Numerical Prediction of Unsteady Flows
- January 18 **Professor Lawrence Snyder**, Purdue University: The Chip Project



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|----------|----|-------------------------------------------------------------------------------------------------------------------------------------------------|
| January  | 20 | <b>Dr. Dennis de Champeaux, Tulane University:</b> Symbolic Evaluation of LISP Functions with Side Effects for Verification                     |
| January  | 25 | <b>Professor Alkesh Punjabi, College of William and Mary:</b> Catastrophes in Elmo Bumpy Torus                                                  |
| January  | 27 | <b>Dr. Charles L. Lawson, Jet Propulsion Laboratory:</b> Interpolation to Scattered Data Over a Sphere                                          |
| February | 10 | <b>Professor Daniel A. Reed, Purdue University:</b> Multimicrocomputer Networks                                                                 |
| February | 11 | <b>Professor Christopher Anderson, University of California, Berkeley:</b> A Vortex Method for Variable Density Flows                           |
| February | 17 | <b>Dr. Eitan Tadmor, ICASE:</b> Quasiconservative Schemes for the Euler Equations                                                               |
| February | 23 | <b>Hillel Tal-Ezer, Tel-Aviv University:</b> Spatial and Temporal Spectral Approximation for Hyperbolic Problems                                |
| February | 24 | <b>Dennis Gannon, Purdue University:</b> A Highly Parallel Data Driven Architecture for Adaptive Finite Element Algorithms                      |
| February | 25 | <b>Dr. Urmila Ghia, University of Cincinnati:</b> On Elliptic Grid Generation Techniques for Arbitrary Geometries                               |
| February | 25 | <b>Dr. Kirti Ghia, University of Cincinnati:</b> A Time Dependent Implicit Technique for the Solution of Incompressible Navier-Stokes Equations |
| March    | 24 | <b>Dr. H. O. Kreiss, California Institute of Technology:</b> Numerical Methods for Stiff Differential Equations                                 |

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